

Economics 742 Lecture 8: Heterogeneous Agent New Keynesian Models Introduction and Estimation

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¹These slides build on excellent slides by Matt Rognlie and Ludwig Straub.

What Is HANK?

- Heterogeneous Agent New Keynesian Models combine:
 1. Heterogeneous Agent Models
 - Bewley-Huggett-Aiyagari and Krusell-Smith
 - Idiosyncratic income risk, precautionary savings motives, credit constraints, et cetera.
 2. The New Keynesian Model
 - Best model we have for thinking about monetary policy.
 - Fun Summary Of Optimal Policy Literature:
https://diercks.shinyapps.io/anthony_diercks_shiny_app/
- Why make this combination?

Two Properties of RBC-Style Models

- FOC relating consumption, labor, and productivity

$$f'_t(n_t) \times u'(c_t) = v'(n_t)$$

- Investment crowds out consumption and vice versa.
 - Hard to get comovement.
- Inter-temporal Euler Equation for Consumption:

$$u'(c_t) = \beta(1 + r_t) u'(c_{t+1})$$

- In steady state $\beta = \frac{1}{1+r}$.
 - Consumption smoothing, PIH, low MPCs.

Two Properties of RBC-Style Models

- FOC relating consumption, labor, and productivity

$$f'_t(n_t) \times u'(c_t) = v'(n_t)$$

- **New Keynesian model** breaks with sticky prices
- Investment and consumption can co-move.
Creates a role for Monetary policy.
- Inter-temporal Euler Equation for Consumption:

$$u'(c_t) = \beta(1 + r_t) u'(c_{t+1})$$

- **Heterogeneous Agent Models** break: Uninsured risk and credit constraints
- High MPCs, limited consumption smoothing.
- Can see space for a synthesis.

How Does HANK Change Macro?

1. Changes the transmission of aggregate shocks
 - May change aggregate response or just transmission.
 - May mean monetary policy makers have to pay attention to features of the economy they previously ignored.
 - Potential state dependence.
2. New shocks or policies can be studied that require heterogeneity to be meaningful.
 - E.g. debt related things, Mortgages, redistribution, etc.
3. Welfare analysis and optimal policy
 - May be completely different despite similar aggregate effects.
 - Most nascent part of this literature because need to think about optimal policy in set of models largely solved on computer.

Four Forces Behind HANK

1. Theory

- New Keynesian model has reached maturity.
- Heterogeneous Agents literature progress in light of new data.

2. Micro Data

- MPCs: Higher than in representative agent model.
- Income distribution and income risk.
- Wealth distribution and household balance sheets.

3. Great Recession: Mian and Sufi on household balance sheets and deleveraging; need Heterogeneous Agents to study.

4. Computation

- Computing Power.
- New Methods.

New Applied Work on High MPCs

- Prior benchmark: Krusell-Smith *approximate aggregation*.
 - Aiyagari with agg shocks looks like rep agent.
 - Intuition: Constrained agents also small part of aggregates. Also, strong incentive to save out of credit constraints; in equilibrium not enough constrained agents to matter.
- Recent introduction of models where Krusell-Smith approximate aggregation fails because large number of high MPC individuals as in data.
 - E.g. Kaplan-Violante on “wealthy hand to mouth” in two-asset models.
- This leads directly to HANK.
 - But NK models had hand to mouth for a while.
 - Question of what this adds always at forefront (HANK vs. TANK).
 - Message is very “back to the future.”

HANK: Back to the Future?



Olivier Blanchard
@ojblanchard1



Soon, we shall have all the channels that the 1970s macro econometric models had captured. 😞 I am struck by the similarity of this graph to a graph I remember and which explained the monetary transmission mechanism in the MPS model built by Modigliani and others.



Andrea Presbitero @a_presbitero · Sep 6

This one should be shown to all bashers of economics who write simplistic criticisms, assuming all macro is still based on the representative agent. #whateconomistsreallydo [twitter.com/ben_moll/statu...](https://twitter.com/ben_moll/status...)

9:22 AM · Sep 7, 2019 · [Twitter Web App](#)

58 Retweets **202** Likes

HANK: Back to the Future?

🔄 Ben Moll Retweeted



Olivier Blanchard
@ojblanchard1



Some good points made re tweet below. Relative to 1970s, we now have a wealth of micro data to characterize each relation more accurately. This can and should be done in effect equation by equation, without feeling that it has to be put back into the big model right away.



Olivier Blanchard @ojblanchard1 · Sep 7

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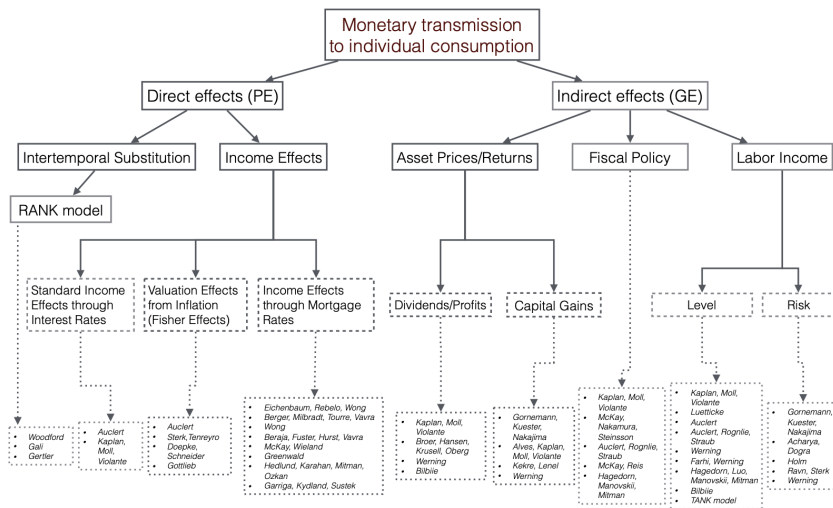
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9 Retweets 42 Likes

Computational References

- Our exploration of the HANK literature will focus on the economics and literature, not the computation.
- But if you want to do research in this area you will need to tech up and learn lots of computational methods!
- General Computational Resources
 - First Computational Book: Miranda and Fackler
Applied Computational Economics and Finance
 - The Computational Bible (Albeit a Bit Old):
Judd *Numerical Methods in Economics*
 - Quantecon.org: Lectures, Python and Julia libraries and tutorials, etc.
- More HANK-Specific Resources
 - Alisdair McKay Short Courses on Numerics and Computation for HANK at alisdairmckay.com
 - Ben Moll's Code Database at benjaminmoll.com
 - Ahn et al. (2018, Macro Annual) Toolbox on GitHub and Paper
 - Auclert et al. (2021, Ecma) Toolbox on GitHub and Paper
- Would suggest you learn Python or Julia.

Ben Moll's HANK Taxonomy



Note: A bit dated so not all our papers are on here.

HANK Section Outline

1. Lecture 1: Introduction to HANK and Estimation
2. Lecture 2: Inspecting the Mechanism: Redistribution and Incomplete Markets
3. Lecture 3: Empirical Evidence on HANK
4. Reading Group 1: Fiscal-Monetary Interaction in HANK
5. Reading Group 2: Behavioral HANK
6. Reading Group 3: Investment, Labor Market, and Misallocation in HANK
7. Reading Group 4: Mortgages and Durables in HANK

For Our HANK Reading Group

- DO THE READING.
 - If you cannot read both (or in a few classes all three) papers, read one carefully and read the intro of the other.
 - Come prepared with a thoughtful comment on the paper you read closely.
 - This will be a discussion! If you are not giving your opinions, it will not be as useful.
- I suggest reading the Kaplan and Violante JEP paper on HANK. It's an easy read and helps set the stage.
 - Also McKay and Wolf (2023) JEP paper, Auclert et al. (2025) AR paper.
- Remember to send me your slides at least 48 hours before.
 - Your slides should focus on the main paper in depth but also cover the related papers at the back of the syllabus to give us the context of the broader literature.

Outline For Today

- Today I will be focusing on two of the key papers in the literature and one review article:
 1. Kaplan Moll Violante “Monetary Policy According to HANK” *AER* 2018
 2. Auclert, Rognlie, and Straub “Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model.” WP 2020 R&R *AER*
- These papers are closely related.
 - KMV launches the literature. It’s analogous to the papers that codified the 3-equation NK model.
 - ARS create a medium scale version that matches hump-shaped IRFs. It’s analogous to Christiano-Eichenbaum-Evans and Smets-Wouters for HANK.
- Also ARS “Fiscal and Monetary Policy With Heterogeneous Agents” in *Annual Review*
 - Good reference and includes some “fixes”

PART 1: Kaplan, Moll, and Violante

“Monetary Policy According to HANK”

KMV Outline

- You saw this paper in 704, so I want to focus on things that you probably missed as a first year.
 - What did you think this time?
 - How were your reactions different?

KMV Outline

- You saw this paper in 704, so I want to focus on things that you probably missed as a first year.
 - What did you think this time?
 - How were your reactions different?
- 1. Paper summary
 - HA: Heterogeneous and on average larger MPCs.
 - NK: Aggregate demand effects.
- 2. What does HANK change?
- 3. What matters for the strength of MP transmission in HANK?
- 4. Why not a one-asset model?
- 5. How does HANK compare to TANK? Is it necessary?
- 6. Do we believe the assumptions that make Fiscal Policy powerful?
- 7. Where should the literature go from KMV?

KMV Summary: Direct vs. Indirect Effects

- Decomposition into substitution and income effects:

$$dC_0 = \underbrace{\int_0^{\infty} \frac{\partial C_0}{\partial r_t} dr_t dt}_{\text{Direct Response to } r} + \underbrace{\int_0^{\infty} \frac{\partial C_0}{\partial Y_t} dY_t dt}_{\text{Indirect Effects Due to } Y}$$

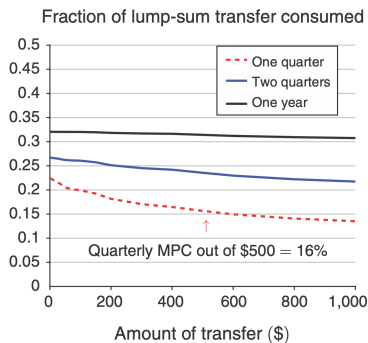
- RANK: >90% direct effects through intertemp sub.
 - Change in r leads to small change in permanent income.
- HANK: direct effects are <20%, indirect >80%.
 - High MPC \rightarrow wealth effects matter.
 - Work through wages, transfers due to government BC, illiquid asset returns and portfolio balancing.
 - More constrained agents \rightarrow fewer on Euler \rightarrow intertemporal substitution affects fewer agents.

KMV: Changed Monetary Transmission Mechanism

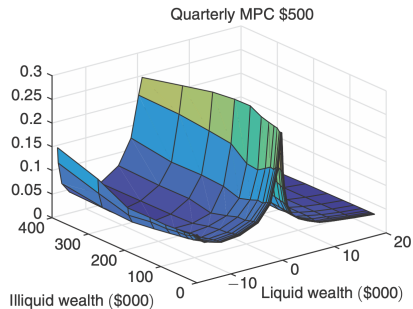
1. Monetary policy works through Central Bank's ability to move labor demand and put money in household pockets.
 - Anything that weakens pass-through to household labor income limits monetary transmission.
 - Works through fiscal redistribution, labor demand created by investment boom, and amplified direct effects.
 - Lots more for central banks to think about!
2. Because of failure of Ricardian equivalence, potency of monetary policy intertwined with fiscal response.
 - Monetary policy relaxes government budget constraint.
 - Timing and distribution of the fiscal response is crucial (assume lump-sum rebated immediately in baseline).
3. Strength depends on household asset distributions and precisely who gets income generated by shifts in labor demand.

MPC Heterogeneity in HANK

Panel A. $\int MPC_{\tau}^x(a, b, z) d\mu$ by τ, x



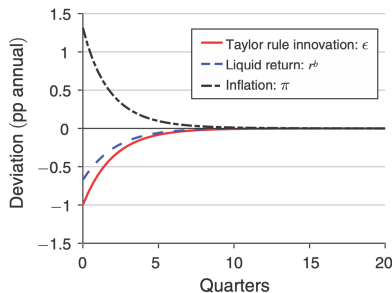
Panel B. $MPC_1^{500}(a, b, z)$



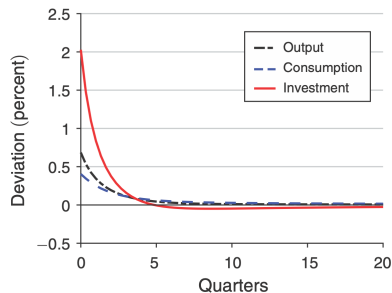
HANK: Impulse Response to Monetary Shock

- Impulse response to decline in interest rates.
 - Lump sum transfers adjust to keep budget balanced in baseline.
 - Slightly stronger transmission than RANK in this case.

Panel A. Monetary shock, interest rate, inflation

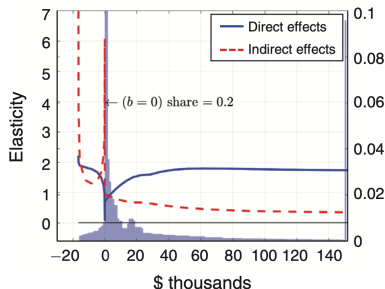
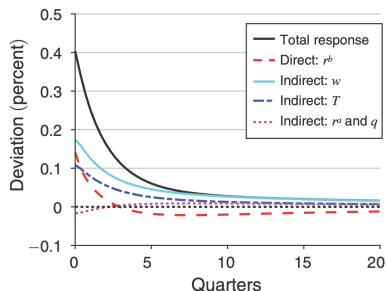


Panel B. Aggregate quantities



HANK: Decomposition of Mon Policy

- Direct effects $\approx 20\%$ of overall response. $\approx 80\%$ indirect.



HANK: Interaction with Fiscal Policy

- Transmission of monetary policy is similar if T , G , or τ adjusts, but weaker if government debt / budget deficit adjusts because pass-through to labor demand or transfers is weakened.

TABLE 8—IMPORTANCE OF FISCAL RESPONSE TO MONETARY SHOCK

	T adjusts (1)	G adjusts (2)	τ adjusts (3)	B^g adjusts (4)
Elasticity of C	-2.93	-2.80	-2.75	-1.68
Partial eq. elasticity of C	-0.55	-0.60	-0.56	-0.71
<i>Component of percent change in C due to</i>				
Direct effect: r^b	19	21	20	42
Indirect effect: w	51	81	62	49
Indirect effect: T	32	—	—	9

Implications for Optimal Policy

1. Policy instrument is less direct and relies more on equilibrium feedbacks.
 - More for the CB to think about. Investment, labor markets, financial markets, etc.
2. Transitory vs. Persistent Rate Cuts:
 - In RANK, transitory-but-large vs. persistent-but-small rate cuts have the same effect.
 - In HANK a transitory but large cut can be more effective: larger reduction in interest payments \Rightarrow more fiscal stimulus.
3. Inflation-Output Tradeoff Depends on Fiscal Response:
 - Phillips curve pinned down by NK side in RANK and HANK, which are the same, so similar slope.
 - Fiscal response matters for slope. More passive fiscal response \Rightarrow less non-neutrality \Rightarrow more favorable CB trade-off.

What Does Strength of HANK Channels Depend On?

1. How profits are distributed and whether illiquid or liquid.
 - If fully illiquid, consumption response weaker but investment decline when output rises at odds with data.
 - If paid as dividends, consumption responds a lot due to huge reaction of investment.
 - So add parameter $\omega = .33$ that controls fraction of profits paid out as dividends vs. reinvested in illiquid account.
 - Always struck me as odd band-aid to model.
2. Income effect
 - GHH preferences strengthen HANK. Initial paper had >95% indirect with GHH.
3. Fiscal response
 - Key is how much high MPC agents get.
 - Fiscal policies that target high MPC agents make MP stronger.

What Does Strength of HANK Channels Depend on?

TABLE 7—DECOMPOSITION OF THE EFFECT OF MONETARY SHOCK ON AGGREGATE CONSUMPTION

	Baseline (1)	$\omega = 1$ (2)	$\omega = 0.1$ (3)	$\frac{\varepsilon}{\theta} = 0.2$ (4)	$\phi = 2.0$ (5)	$\frac{1}{\nu} = 0.5$ (6)
Change in r^b (pp)	-0.28	-0.34	-0.16	-0.21	-0.14	-0.25
Elasticity of Y	-3.96	-0.13	-24.9	-4.11	-3.94	-4.30
Elasticity of I	-9.43	7.83	-105	-9.47	-9.72	-9.79
Elasticity of C	-2.93	-2.06	-6.50	-2.96	-3.00	-2.87
Partial eq. elasticity of C	-0.55	-0.45	-0.99	-0.57	-0.59	-0.62
<i>Component of percent change in C due to</i>						
Direct effect: r^b	19	22	15	19	20	22
Indirect effect: w	51	56	51	51	51	38
Indirect effect: T	32	38	19	31	31	45
Indirect effect: r^a and q	-2	-16	15	-2	-2	-4

- Payout policy (ω) *really* matters.
- Taylor rule coefficient ϕ and price stickiness θ mostly affect inflation response.
- Lower Frisch $1/\nu$ shifts from wage to transfer; elast unchanged.

Why Not a One-Asset Model?

- In one-asset models, there is a tension between matching high observed wealth/output and generating a large average MPC.
- Intuitively, you need wealthy hand to mouth and you only get poor hand to mouth with one asset.
- By matching dist of MPCs in data, two-asset model makes high MPC from HA part of HANK quantitatively significant.
 - However KMV show you can get this with a high discount rate as long as you miss overall level of wealth in the economy.

HANK vs. TANK

- KMV: Total consumption elasticity in TANK is somewhat smaller and share of direct effects is $3\times$ smaller than in HANK
 - Quantitative argument for why HANK is better: Direct effects too strong because so many savers.
 - Sub effect dampened in HANK by prospect of hitting borrowing constraint in future shortening effective planning horizon.

HANK vs. TANK

- KMV: Total consumption elasticity in TANK is somewhat smaller and share of direct effects is $3\times$ smaller than in HANK
 - Quantitative argument for why HANK is better: Direct effects too strong because so many savers.
 - Sub effect dampened in HANK by prospect of hitting borrowing constraint in future shortening effective planning horizon.
- I am skeptical of quantitative argument.
 - They calibrate to 30% spenders and 70% savers based on micro data.
 - If raise hand to mouth share, do better.

Debortoli and Galí (2018): Arguing for TANK

- Three channels through which HANK and TANK differ:
 1. Changes in average C gap between constrained and unconstrained.
 2. Variations in C dispersion within unconstrained due to changing wealth.
 3. Changes in share of unconstrained due to how often borrowing constraint binds.
- Argue 2 and 3 mutually offset. TANK does well b/c it captures changes between constrained and unconstrained groups.
 - DG: TANK does well quantitatively in capturing output dynamics of canonical HANK.
 - But does it do well in all extensions?
- Big debate: Are TANK models “good enough”?
 - TANK more tractable.
 - But HANK people value ability to calibrate to micro data, dislike hard-wiring the “spender-saver” share into the model
 - Your thoughts?

KMV: Final Questions

1. Do we believe the assumptions that make Fiscal Policy powerful?
 - Will return to this with Auclert-Rognlie-Straub.
2. Where should the literature go from KMV?

KMV: Final Questions

1. Do we believe the assumptions that make Fiscal Policy powerful?
 - Will return to this with Auclert-Rognlie-Straub.
2. Where should the literature go from KMV?
 - Estimated models, optimal policy, ZLB.
 - Aggregate shocks, state dependence (out of steady state IRFs).
 - What is “mystery meat” illiquid asset? Do we add anything by modeling it?
 - Gross asset positions instead of net; asset pricing issues.
 - Other frictions.

ARS (Annual Review): Literature Summary

- The ARS annual review article presents a summary of the literature as of December 2024.
 - Excellent review article.
- “Canonical HANK model” with a key change: Sticky wages and flexible prices instead of flexible wages and sticky prices.
 - Simplified version of ARS’ “Intertemporal Keynesian Cross” paper, details there.
 - Provide you with a sketch of how this works here.
- Why?
 1. Large countercyclicality of profits makes models implausibly sensitive to their distribution.
 2. Does not require endogenous labor supply, which leads to implausible marginal propensities to earn.
 - Sticky wages fixes both: low marginal propensity to earn and no profits to allocate.

ARS (Annual Review): Sticky Wages

- Households: Discount factor β_{it} and productivity e_{it} with a_{it} in mutual fund that returns r_t^p and wage per efficiency unit w_t :

$$\max_{c_{it}} E_0 \left[\sum_{t=0}^{\infty} \left(\prod_{s \leq t-1} \beta_{is} \right) [\log(c_{it}) - v(n_{it})] \right]$$

$$s.t. \quad c_{it} + a_{it} \leq (1 + r_t^p) a_{i,t-1} + (1 - \tau_t) w_t e_{it} n_{it}$$

$$a_{it} \geq \underline{a}$$

- Generalize Erceg, Henderson, and Levin (2000) to heterogeneous agents.
 - Labor inputs of type $k \in [0, 1]$ each represented by a union which supplies $N_{kt} = \int e_{it} n_{kit} di$ units of labor.
 - “Labor packer” firm aggregates $k \in [0, 1]$ labor inputs into final labor input using CES tech: $N_t = \left(\int N_{kt}^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}}$.
 - Final output $Y_t = N_t$ (can generalize).

ARS (Annual Review): Sticky Wages

- Assume allocate hours uniformly and max utility of agent with average consumption C_t and discount factor $\bar{\beta}$.
 - Assuming equal allocation so $n_{it} = n_{kit} = N_t$.
- What happens without stickiness?
 - Stripping away heterogeneity, get:

$$v'(N_t) = \frac{\varepsilon - 1}{\varepsilon} (1 - \tau) u'(C_t) = \frac{1 - \tau_t}{\mu C_t}$$

- If plug in for v , get real wage is a markup over MRS.
 - Markup because union is sole seller of type k labor.
- No product market power (so real wage = MPL) \Rightarrow markup creates wedge between MRS and MPL.
 - Equilibrium conditions look similar to product market power case, but with no profits.
 - Prices are equal to $MC = w$. So price dynamics/markup inherit wage dynamics/markup.

ARS (Annual Review): Sticky Wages

- Sticky wages: Unions set wages subject to each wage change incurring a Rotemberg utility cost:

$$\frac{\psi}{2} \int \left(\frac{w_{kt}}{w_{kt-1}} - 1 \right)^2 dk.$$

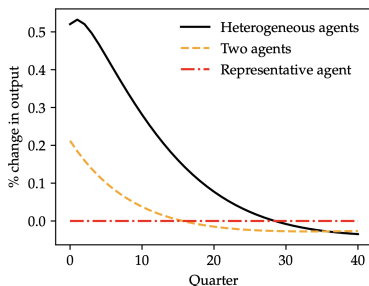
- Equivalent to Calvo to first order but no price dispersion, saving a firm-side state variable, and works in continuous time.
- Obtain wage Phillips curve:

$$\pi_t^w = \kappa \left(v'(N_t) - \frac{1 - \tau_t}{\mu C_t} \right) + \bar{\beta} \pi_{t+1}^w$$

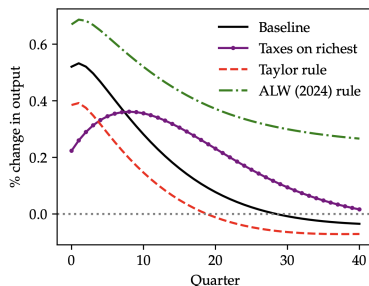
- Term in brackets is the average wedge in the labor FOC
- Wage inflation occurs when MC to agents of working more, v' , exceeds MB from working equal to post tax wage, $\frac{1 - \tau_t}{\mu}$, times MU of consumption $\frac{1}{C}$.
- Prices are constant markup but flexible, so wage PC determines price inflation as well.

ARS (Annual Review): Fiscal Policy

- Key finding: If HANK, RANK, and TANK are all calibrated to same balanced-budget fiscal multipliers, get very different implications for deficit-financed fiscal policy.



(a) Role of heterogeneity

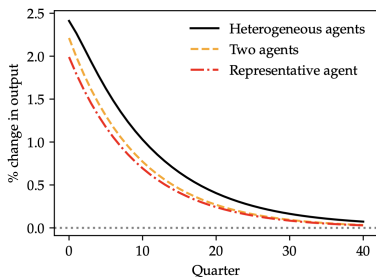


(b) Alternative het.-agent models

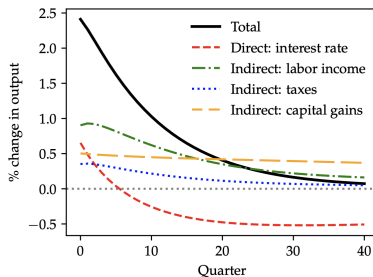
- Intuition: Tax cut is partially spent but also partially saved. Households gradually spend down savings resulting in additional consumption-income cross feedback.

ARS (Annual Review): Monetary Policy

- Key finding: HANK, RANK, and TANK are more similar for monetary policy.



(a) Role of heterogeneity



(b) Decomposition

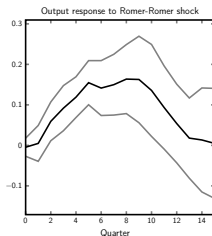
- Intuition: Higher MPC agents have less direct sensitivity to interest rates, which offsets stronger amplification of indirect.
 - Less sensitive to model assumptions than one might expect.

PART 2: Auclert, Rognlie, and Straub

“Micro Jumps Macro Humps”

Goal: Match Micro and Macro of Monetary Policy

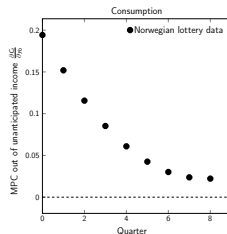
- “Macro Time-Series Approach” (e.g. CEE, SW) seeks to match hump-shaped impulse responses in aggregates.
 - Representative agent (RA) model using habits, adjustment costs, inattention, etc.



- “Micro Moments Approach” (e.g. KMV) seeks to match micro “jumps” (MPCs)
 - Heterogeneous agents (HA) model with income risk and incomplete markets.
- This paper unifies the two approaches and revisits monetary transmission mechanism, sources of business cycles.

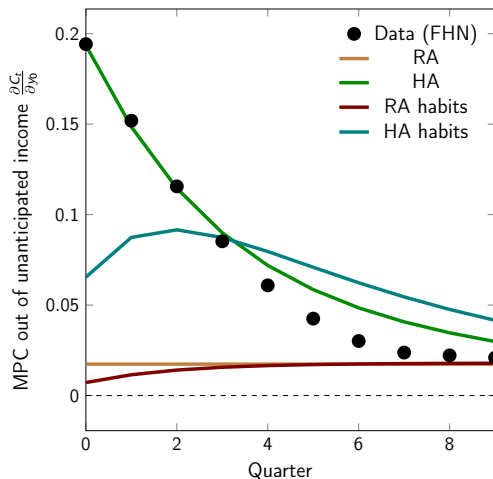
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Heterogeneous Agents Models Match Intertemporal MPCs



Inattention Preserves (i)MPCs But Introduces Sluggishness

- Introduce aggregate risk in r_t, y_t .
- Approach to humps: **sticky expectations**.

[Gabaix-Laibson 2001, Mankiw-Reis 2002, 2006,
Carroll-Crawley-Slacalek-Tokuoka-White 2018]

- agents update expectations w/ Calvo $1 - \theta$; if $k = \#$ periods since last update:

$$V_t(\ell, s; k) = \max_{c, \ell'} u(c) + \beta \mathbb{E}_{t-k} \left[\begin{array}{l} \theta V_{t+1}(\ell', s', k+1) + \\ (1-\theta) V_{t+1}(\ell', s', 0) \mid s \end{array} \right]$$

$$c + \ell' \leq (1 + r_t)\ell + y_t e(s), \ell' \geq 0$$

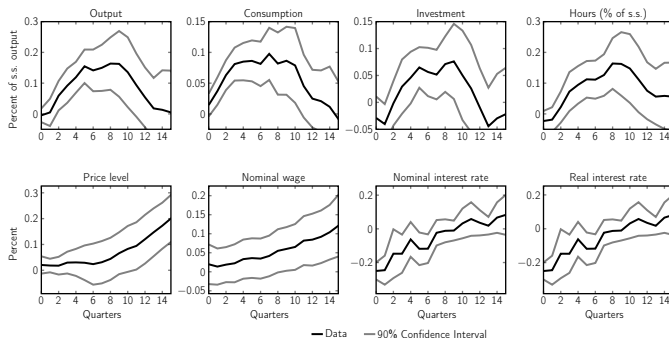
- Agents see current $r_t, y_t \Rightarrow$ never violate borrowing constraint!
- **Achieves two goals:**
 1. (i)MPCs same around the s.s. \rightarrow matches “**micro jumps**”
 2. Beliefs about *future* path of aggregates sluggish
 \rightarrow matches “**macro humps**”

GE Model Overview

- Discrete time with aggregate shocks
- Household side: Two assets and **sticky expectations**
 - Wealth held by competitive and attentive intermediary holding liquid assets with rate $r_t^\ell = r_{t-1} - \xi$ and illiquid assets a_t with return r_t^a .
 - Households are inattentive also w.r.t value of a_t^{illiq} with same Calvo parameter θ for aggregates.
- Supply side: Standard NK with investment adjustment costs, nominal rigidities with indexation.
- Fiscal rule changing labor taxes.
- Monetary policy with inertial Taylor rule.

Estimation

- Split parameters into two categories:
 - Steady-state relevant parameters [income process, liquid asset share, ...]
→ Calibrate to micro, e.g. income distribution, MPCs
 - Impulse-response relevant parameters $\theta, \phi, \zeta^p, \zeta^w, \rho^m, \sigma^m$
→ Estimate to match IRF to monetary policy shocks



How to Simulate and Estimate? Sequence Space Jacobian

- Computational advance from Auclert-Bardoczy-Rognlie-Straub (2021, Ecma).
- Use “**sequence-space Jacobian**” method.
 - Quick summary here.
- Key Innovation: Linearize model in **sequence space**, which is *state of perfect-foresight sequences of aggregate variables*.
 - Unlike state space, *size of system is independent of degree of heterogeneity*.
 - So can compute models with rich heterogeneity at low computational cost.
- Method has taken over computational macro because it is so fast and powerful.
 - Used to take forever to solve models.
 - Now for a broad class of models so fast you can estimate them!

How to Simulate and Estimate? Sequence Space Jacobian

- Want to compute IRF to perfect foresight “MIT shock”
 - Exploit **aggregate certainty equivalence**:
Up to first order identical to stochastic shock.
- **Sequence space Jacobians summarize GE of model.**
 - Ex: Standard incomplete markets model features Jacobian that maps changes in sequence $\{r_t\}$ into $\{C_t\}$, $J^{C,r}$.
 - Heterogeneous responses to r_t and changes in dist across states is “under the hood” in $J^{C,r}$, which is a sufficient statistic.
 - Truncate to T periods, compute Jacobians, and use for IRFs.
- Equilibrium can generically be written as:

$$F(\{X_s, Z_s\}) = 0$$

where $\{Z_s\}$ are exogenous params/shocks.

→ Impulse response:

$$d\mathbf{X} = -F_X^{-1} \cdot F_Z \cdot d\mathbf{Z}$$

- Get F_X and F_Z by combining **model blocks**.
- Methods to **efficiently compute** F_X and F_Z (“fake news”).
- **Code base available online.**

How to Simulate and Estimate?

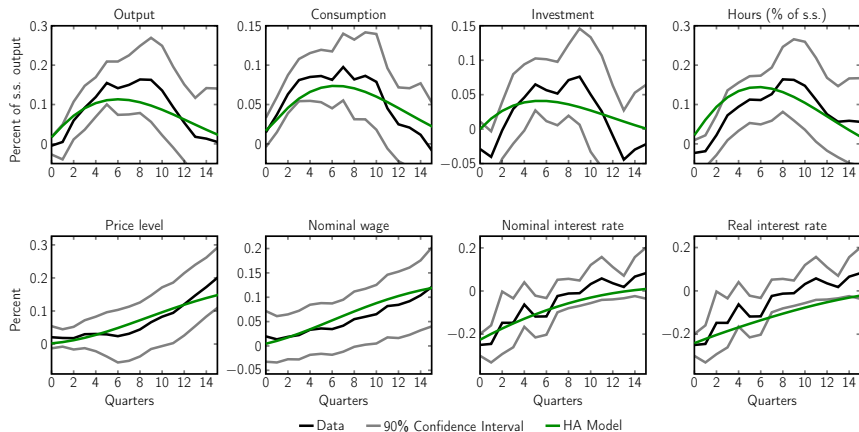
1. Use certainty equivalence \rightarrow focus on small “MIT shocks”
2. Compute **each block's Jacobians** [sufficient for simulation!]:

$$\begin{pmatrix} dC_0 \\ dC_1 \\ dC_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & \cdots \\ M_{10} & M_{11} & M_{12} & \cdots \\ M_{20} & M_{21} & M_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} dY_0 \\ dY_1 \\ dY_2 \\ \vdots \end{pmatrix} + \dots$$

$$\begin{pmatrix} dC_0 \\ dC_1 \\ dC_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} M_{00} & (1-\theta)M_{01} & (1-\theta)M_{02} & \cdots \\ M_{10} & (1-\theta)M_{11} + \theta M_{00} & (1-\theta)M_{12} + \theta(1-\theta)M_{01} & \cdots \\ M_{20} & (1-\theta)M_{21} + \theta M_{10} & \vdots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- With sticky expectations, manipulate standard Jacobian!
Weight by prob of having last updated at time s , and
propagate non-updaters' time- s policy functions forward.

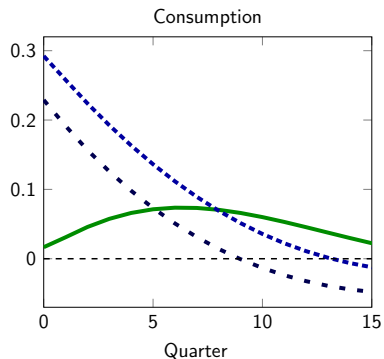
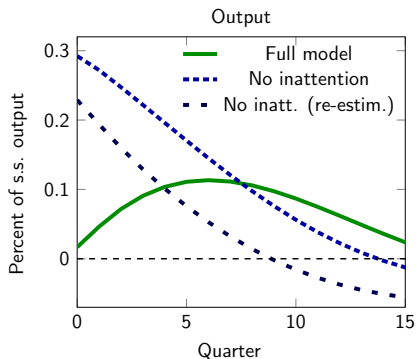
Estimated IRFs



Estimates point to significant inattention

Estimated parameters			
Parameter		Value	std. dev.
θ	Stickiness of household expectations	0.935	(0.01)
ϕ	Investment adjustment cost	9.639	(2.428)
ζ^p	Calvo price stickiness	0.926	(0.012)
ζ^w	Calvo wage stickiness	0.899	(0.016)
ρ^m	Taylor rule inertia	0.890	(0.01)
σ^m	Standard deviation of monetary shock	0.057	(0.005)

Sticky Expectations are Crucial For Hump Shape

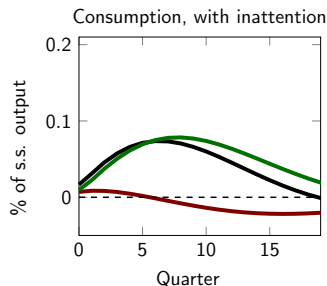
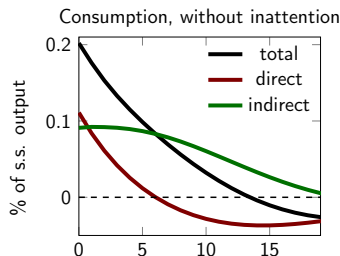


Inattention Dampens Direct Effects

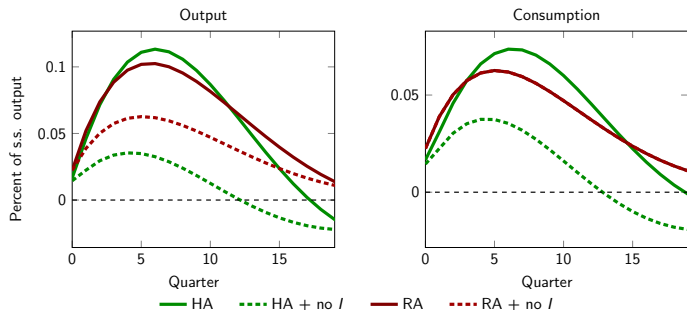
- Decomposition following Auclert, 2018:

$$dC_t = \underbrace{\sum_s \frac{\partial C_t}{\partial r_s} dr_s}_{\text{direct}} + \underbrace{\sum_s \frac{\partial C_t}{\partial Y_s} dY_s}_{\text{indirect}} + \dots$$

- Indirect effects** driven by MPCs, unaffected by attention.
- Direct effects** dampened by inattention → intertemporal substitution plays essentially no role

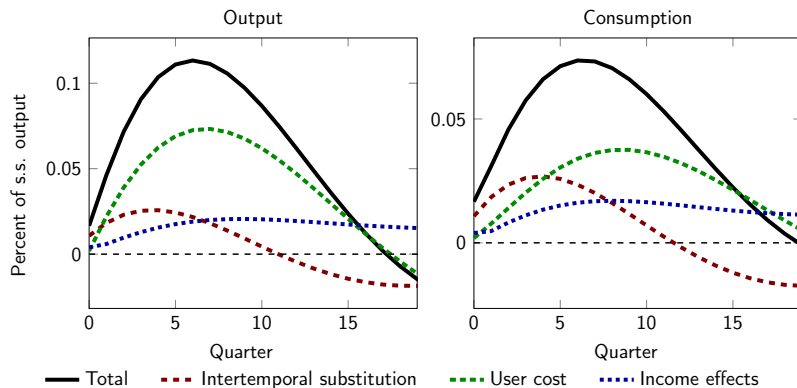


Investment Plays a Crucial Role in Monetary Transmission



- Switching off investment dampens HA Y response by 85% and C by 70%. Does nothing to C in RA.
- Intuition: Without intertemporal substitution, can't initiate Keynesian multiplier; I is main r sensitive object left.
 - What model features are crucial for this? Do we believe it?

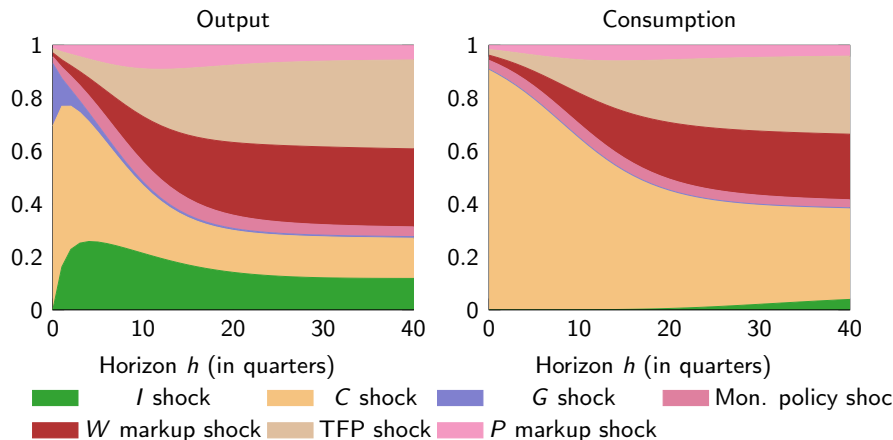
Decomposition of What Gets Monetary Transmission Started



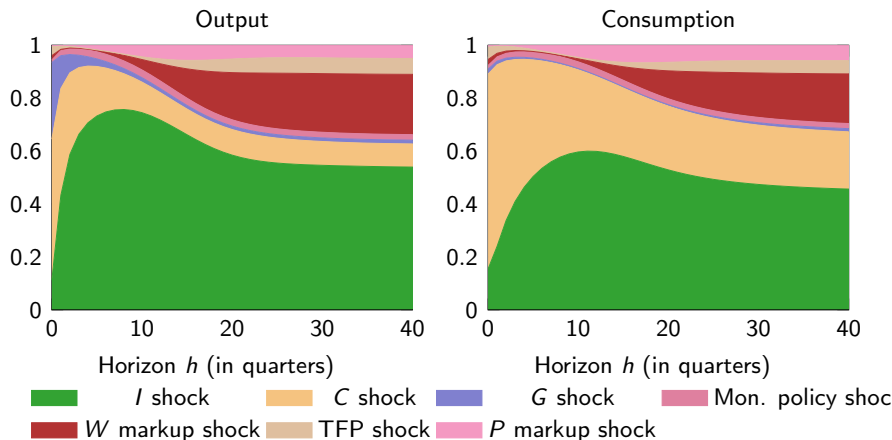
Bayesian Estimation To Decompose Importance of Shocks

- Go full Smets-Wouters!
 - Add 7 shocks (TFP, wage markup, price markup, monetary policy, government spending, consumption, investment).
 - Discount factor shock for C , risk premium shock for I .
 - Use same model parameters but estimate all shock parameters to match 7 standard series
 - Compare to RA with habit (SW)
- Use to assess relative importance of each shock for driving the business cycle.
 - Plot forecast error variances $Var_t(Y_{t+h})$ at business cycle horizons to assess.

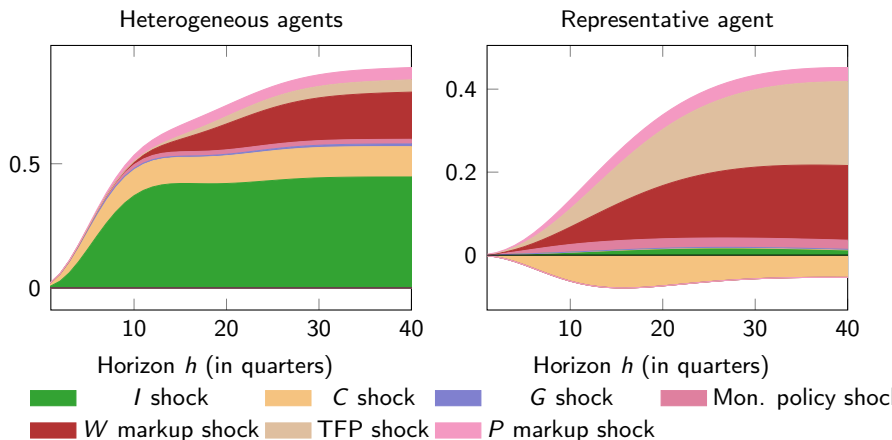
RA FEVD: Markup and TFP Shocks Dominate



HA FEVD: Investment Shocks Dominate

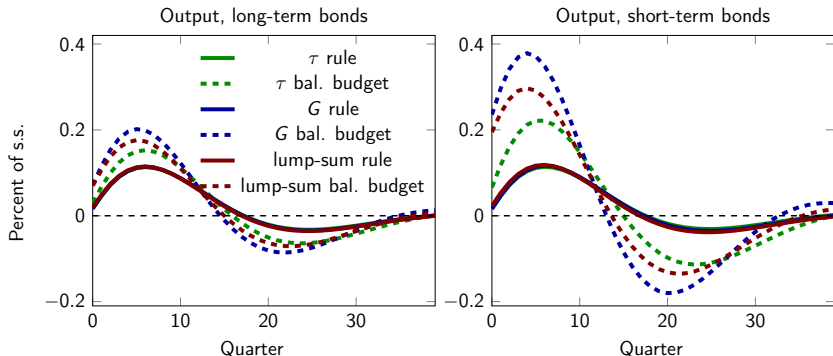


Intuition: Shocks Needed For High $\text{Cov}(C_{t+h}, I_{t+h})$



- HA with inattention \Rightarrow endogenous C - I comovement.
- Due to comovement, I shocks move labor wedge.

Critique of KMV Fiscal Policy Results



- With long-term bonds, much less of a windfall from lower r_t .
 - Precise fiscal rule matters much less than with short-term.
 - I find this compelling!

Evaluation

- This is my favorite recent HANK paper.
- I believe main message that matching macro humps dampens intertemporal substitution dramatically and makes investment a crucial transmission mechanism.
- Main criticism: Investment is the last remaining suspect *in the model*. Is this true in the real world?
 - E.g. Monetary transmission through housing and mortgage refinancing or durable purchases.
 - No real direct evidence for investment. It's really that intertemporal substitution does relatively little so it does a lot.
- Investment adj cost is lower in HA than RA. Is this important?
 - In this literature, shape of adjustment costs matters a *lot*. What role does this play?
- Don't need heterogeneity to kill Barro-King result that investment and consumption do not endogenously co-move. Just need inattention.